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Midly Mappings in Fuzzy Topological Spaces

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Abstract: In this paper, we introduce fuzzy mildly Minimal-open and fuzzy mildly Minimal-closed functions in fuzzy topological spaces and obtain certain characterization of the mildlyMinimal closed and mildly Minimal open functions.

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I. Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A: X \longrightarrow [0,1]$ of A, defined by
$\mu_A(x) = 1$, if $x \in A$.
$= 0$, if $x \in A$.

Thus an element $x \in X$ isin A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)) : x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only.

Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function $\mu_A : X$ [0,1] , which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy mildlyMinimal-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy mildlyMinimal-closed but not conversely. Also we introduce fuzzy mildlyMinimal-open sets in fuzzy topological spaces and study some of their properties.

1. PRELIMINARIES

1.1 Definition: [1] A fuzzy subset A in a set X is a function A: X → [0, 1]. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by $\mu_x$or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Definition : [1] If A and B are any two fuzzy subsets of a set X , then A is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.3 Definition:[1]Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X. Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.4 Definition:[1] The complement of a fuzzy subset A in a set X, denoted by A′ or 1 − A, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that$(A')' = A$.

1.5 Definition:[1] The union of two fuzzy subsets A and B in X, denoted by A ∨ B, is a fuzzy subset in X defined by $(A \vee B)(x) = \text{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.6 Definition:[1]The intersection of two fuzzy subsets A and B in X, denoted by A ∧ B, is a fuzzy subset in X defined by $(A \wedge B)(x) = \text{Min}\{A(x), B(x)\}$ for all x in X.

1.7 Definition:[1] A fuzzy set on X is ‘Crisp‘ if it takes only the values 0 and 1 on X.

1.8 Definition:[2]Let X be a set and be a family of fuzzy subsets of X. τ is called a fuzzy topology on X iff τ satisfies the following conditions.

(i)μ_ϕ ; $\mu_X \in \tau$: That is 0 and 1 $\in \tau$

(ii) If $G_i \in \tau$ for $i \in I$ then $\bigvee_{i \in I} G_i \in \tau$

(iii) If $G,H \in \tau$ then $G \wedge H \in \tau$

The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.9 Remark:[2] Every topological space is a fuzzy topological space but not conversely.

1.10 Definition:[2]Let X be a fts and A be a fuzzy subset in X. Then $\wedge$ {B : B is a closed fuzzy set in Xand B $\geq$ A} is called the closure of A and is denoted by A or cl(A).

1.11 Definition:[2]Let A and B be two fuzzy sets in a fuzzy topological space (X,τ)and let A $\geq$ B. Then B is called an interior fuzzy set of A if there exists G $\in\tau$ suchthat A $\geq$ G $\geq$ B, the least upper bound of all interior fuzzy sets of A is called the interior of A and is denoted by A^0.

1.12 Definition[3] A fuzzy set A in a fts X is said to be fuzzy semi open if and only if there exists a fuzzy open set V in X such that V $\leq$A $\leq$cl(V).

1.13 Definition [3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that int(V)$\leq$ A $\leq$V. It is seen that a fuzzy set A is fuzzy semi open if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem:[3] The following are equivalent:

(a) $\mu$is a fuzzy semiclosed set,
(b) μ^C is a fuzzy semi open set,
(c) int(cl(μ)) $\leq$ μ.
(b) int(cl(μ)) $\geq$ μ^c

1.15 Theorem [3] Any union of fuzzy semi open sets is a fuzzy semi open set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark[3]

(i) Every fuzzy open set is a fuzzy semi open but not conversely.
(ii) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(iii) The closure of a fuzzy open set is fuzzy semi open set
(iv) The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition:[3] A fuzzy set μ of a fts X is called a fuzzy regular open set of X if int(cl(μ).= μ.

1.18 Definition:[3] A fuzzy set μ of fts X is called a fuzzy regular closed set of X if cl(int(μ)) = μ.

1.19 Theorem:[3] A fuzzy set μ of a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.20 Remark:[3]

(i) Every fuzzy regular open set is a fuzzy open set but not conversely.
(ii) Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem:[3]

(i) The closure of a fuzzy open set is a fuzzy regular closed.
(ii) The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition:[4] A fuzzy set αin fts X is called fuzzy rw -closed if cl(α)$\leq$ μ whenever $\alpha \leq$ $\mu$and μ is regular semi-open in X.

1.23 Definition [5]:A fuzzy set αin fts X is called fuzzy mildlyMinimal closed if p-cl(α)$\leq$ μwhenever $\alpha \leq$ $\mu$and $\mu$is rgα $-$open set in X.

1.24 Definition [5]:A fuzzy set α of a fts X is fuzzy mildlyMinimal-open set, if it's complement α^c is a fuzzy mildlyMinimal-closed in fts X.

1.25 definition [2]: Let X and Y be fts. A function f: X $\rightarrow$ Y is said to be a fuzzy continuous functionping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.

1.26 definition[6]:A functionping f: X $\rightarrow$ Y is said to be fuzzy irresolute iff f^{-1} (B) is fuzzy semi-open in X is fuzzy open in Y.

1.27 Definition[7]: A functioning f: X $\rightarrow$ Y is said to be fuzzy mildlyMinimal irresolute if the inverse image of every fuzzy mildlyMinimal-open in Y is a fuzzy mildlyMinimal-open set in X.

2. FUZZY MILDLY MINIMAL-OPEN FUNCTIONS AND FUZZY MILDLY MINIMAL CLOSED FUNCTIONS IN FUZZY TOPOLOGICAL SPACES

Definition 2.1: Let X and Y be two fts. A function f: (K,T1) $\rightarrow$ (L,T2) is called fuzzy mildly Minimal-open function if the image of every fuzzy open set in X is fuzzy mildly Minimal-open in Y.

Theorem 2.2: Every fuzzy open function is a fuzzy mildlyMinimal-open function.
Proof: Let f :(K,T1) $\rightarrow$ (L,T2) be a fuzzy open function and and μ be a fuzzy open set in fts X. Then f(μ) is a fuzzy open set in fts Y. Since every fuzzy open set is fuzzy mildlyMinimal -open,f(μ) is a fuzzy mildlyMinimal-open set in fts Y. Hence f is a fuzzy mildlyMinimal-open function.

The converse of the above theorem need not be true in general as seen from the following example.

Example 2.3: Let X= Y={p, q, r} and the functions α, β: X $\rightarrow$ [0, 1] be defined as

α (x) = 1 if x = p,r
0 otherwise

β(x) = 1 if x = p
0 otherwise

Consider T_1= {1,0,α}, T_2 = {1,0,β}then (K,T1) $\rightarrow$ (Y, T_2) are fts. Let function f: X $\rightarrow$ Y be the identity function. Then this function is mildly Minimal-open function but it is not fuzzy open . Since the image of the fuzzy open set α (x) in X is fuzzy set β in Y which is not fuzzy open.

Remark 2.4 : If f: (K,T1) $\to$ (L,T2) and g:(L,T2) $\to$ (Z,T_3) be two fuzzy mildly Minimal open function then composition gof: (K,T1) $\to$ (Z,T_3) need not be fuzzy mildly Minimal-open as seen from the following example

Example 2.5: Let X = Y = Z = {a,b,c} and the function $\alpha, \beta, \gamma, \delta$: X $\to$ [0,1] be defined as

α (x) = 1 if x = p
0 otherwise

β(x) = 1 if x = r
0 otherwise

γ(x) = 1 if x = p,r
0 otherwise

δ(x) = 1 if x = p,q
0 otherwise

Consider $T_1 = \{1,0,\alpha,\delta\}$, $T_2 = \{1,0,\alpha\}$and $T_3 = \{ 0,1, \alpha, \beta, \gamma\}$. Let function f: (K ,$T_1$) $\to$ (L,T2) and g: (L,T2) $\to$ (Z,T_3) be the identity functions then f and g are fuzzy mildlyMinimal-open function but their composition gof:(K,T1) $\to$ (Z,T_3) is not fuzzy mildlyMinimal-open as β: X $\to$ [0,1] is fuzzy open in X.But (gof)(β) = β is not fuzzy mildly Minimal-open in Z.

Theorem 2.6 : If f: (K,T1) $\to$ (L,T2) is fuzzy open function and g:(L,T2) $\to$ (Z,T_3) is fuzzy mildlyMinimal open function then their composition gof: (K,T1) $\to$ (Z,T_3) is fuzzy mildlyMinimal-open function.

Proof: Let α be fuzzy open set in (K,T1). since f is fuzzy open function(α) is a fuzzy open set in (L,T2).Since g is a fuzzy mildlyMinimal-open function g(f(α))= (gof)(α). Thus gof is a fuzzy mildly Minimal-open function.

Definition 2.7: Let X and Y be two fuzzy topological spaces. A function f:(K,T1) $\to$ (L,T2) is called fuzzy mildlyMinimal-closed function if the image of every fuzzy closed set in X is a fuzzy closed set in Y.

Theorem 2.8: Let f: (K,T1) $\to$ (L,T2)be fuzzy closed function then f is a fuzzy mildlyMinimal closed function.

Proof: Let α be a fuzzy closed set in (K,T1). since f is fuzzy closed function,f(α) is a fuzzy closed function,f(α) is a fuzzy closed set in (L,T2). since every fuzzy closed set is fuzzy mildlyMinimal-closed ,f(α) is a fuzzy mildlyMinimal-closed set in (L,T2).Hence f is fuzzy mildly Minimal-closed function.

The converse of the theorem need not be true in general as seen from the following example

Example 2.9: Let X= Y={a, b, c} and the functions $\alpha, \beta: X \to [0, 1]$ be defined as

α (x) = 1 if x = a,c
0 otherwise

β(x) = 1 if x = a
0 otherwise

Consider T_1= {1,0,α} and T_2= {1,0,β}; then (K,T1) and (L,T2) are fts . Let f: (K,T1) $\to$ (L,T2) be the identity function. Then f is a fuzzy mildly Minimal-closed function but it is not a fuzzy closed function, since the image of the closed set α in X is not a fuzzy closed set in Y.

Remark 2.10: The composition of two fuzzy mildlyMinimal closed functions need not be a fuzzy mildlyMinimal-closed function

Example 2.11: Consider the fts (K,T1),(L,T2)(Z,T_3) and functionpings defined in 2.5. the functions f and g are fuzzy mildlyMinimal-closed but their composition is not fuzzy mildlyMinimal-closed as β:X $\to$ [0,1] is a fuzzy closed set in X but (gof)(β)=β is not fuzzy mildlyMinimal-closed in Z.

Theorem 2.12: If a function f: (X, T1) $\to$ (L,T2) and g: (L,T2) $\to$ (Z,T_3) be two functions then composition of two mildlyMinimal-closed functions is fuzzy mildlyMinimal-closed function.

Proof: Let α be a fuzzy closed set in (K,T1). Since f is a fuzzy closed function, f(α) is a fuzzy closed set in (L,T2). Since g is a fuzzy mildly Minimal-closed function but g(f(α)) is a fuzzy mildly Minimal-closed set in (Z,T_3) but g(f(α))=(gof)(α) .Thus gof is fuzzy mildlyMinimal-closed function.

Theorem 2.13: A function f: X $\to$ Y is fuzzy mildlyMinimal-closed function if for each fuzzy set δ of Y and for each fuzzy open set μ of X s.t $\mu \geq f^{-1}(\delta)$, there is a fuzzy mildly Minimal-open function set α of Y s.t $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$.

Proof: Suppose that f is fuzzy mildlyMinimal-closed function. Let δ be a fuzzy subset of Y and μ be a fuzzy open set of X s.t $f^{-1}(\delta) \leq \mu$. Let α = 1 – f(1- μ) is fuzzy mildlyMinimal-open set in fts Y. note that $f^{-1}(\delta) \leq \mu$.Which implies $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$ .For the converse, Suppose that μ is a fuzzy closed set in X, Then f^{-1}(1-f (μ)≤1- μ And 1- μ is fuzzy open by hypothesis, there is a fuzzy mildly Minimal-open set α of Y s.t (1-f (μ)≤ α .And $f^{-1}(\alpha) \leq 1-\mu$. Therefore $\mu \leq 1- f^{-1}(\alpha)$. Hence 1- α ≤ μ , f(1-$f^{-1}(\alpha)$) ≤ 1- α.Which implies f (μ) = 1- α . Since 1- α is fuzzy mildly Minimal-closed f (μ) is fuzzy mildlyMinimal-closed and thus f is fuzzy mildlyMinimal-closed.

Lemma 2.14: Let f :(K,T1) $\to$ (L,T2) be fuzzy irresolute function and α be fuzzy rgα open in Y then $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Proof: Let α be fuzzy rgα-open in Y. To prove $f^{-1}(\alpha)$ is fuzzy rgα open in X that is to prove $f^{-1}(\alpha)$ is both fuzzy semi-open and fuzzy semi-closed in X.Now α is fuzzy semi-open in Y. Since f is fuzzy irresolute function,$f^{-1}(\alpha)$ is fuzzy semi-open in X.Now α is fuzzy semi-closed in Y as fuzzy rgα open set is fuzzy semi-closed then 1- α isFuzzy semi-open in Y. Since f is fuzzy irresolute function, $f^{-1}(1-\alpha)$ is a fuzzy semi open in X.But $f^{-1}(1-\alpha) = 1 - f^{-1}(\alpha)$ is fuzzy semi-open in X and so $f^{-1}(\alpha)$ is semi closed in

X. Thus $f^{-1}(\alpha)$ is both fuzzy semi-open and fuzzy semi-closed in X and hence $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Theorem 2.15: If a function f: (K,T1) $\rightarrow$ (L,T2) is fuzzy irresolute function and fuzzy mildlyMinimal-closed and α is fuzzy mildlyMinimal-closed set of X,then f(α) is a fuzzy mildlyMinimal-closed set in Y.

Proof: Let α be a fuzzy closed set of X.Let f(α) $\leq \mu$. Where μ is fuzzy rgα open in Y.Since f is fuzzy irresolute function, $f^{-1}(\mu)$ is a fuzzy rgα open in X, by lemma 2.14 and $\alpha \leq f^{-1}(\mu)$. Since α is a fuzzy mildlyMinimal-closed set in X,P-cl(α) $\leq f^{-1}(\mu)$. Since f is a fuzzy mildlyMinimal-closed set contained in the fuzzy rgα open set μ, which implies cl(f(p-cl(α)) $\leq \mu$ and hence cl(f(α)) $\leq \mu$. Therefore f(α) is a fuzzy mildlyMinimal closed set in Y.

Corollary 2.16: If a function f: (K,T1) $\rightarrow$ (L,T2) is fuzzy irresolute function and fuzzy closed and α is a fuzzy mildly Minimal-closed set in fts.

Proof : The proof follows from the theorem 2.15 and the fact that every fuzzy closed function is a fuzzy mildlyMinimal-closed function.

Theorem 2.17: Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two functioning's s.t

(i) if f is fuzzy continuous function and subjective, then g is fuzzy mildlyMinimal closed function.
(ii) if g is fuzzy mildlyMinimal-irresolute function and injective then f is fuzzy mildlyMinimal-closed function.

Proof :

(i) Let μ be a fuzzy closed set in Y ,since f is fuzzy continuous function,$f^{-1}(\mu)$ is a fuzzy closed set in X. Since gof is a fuzzy mildlyMinimal closed function,(gof)($f^{-1}(\mu)$) is a fuzzy mildlyMinimal-closed set in Z. but (gof) ($f^{-1}(\mu)$) = g(μ) as f is surjective thus g is fuzzy mildlyMinimal closed function.

(ii) Let β be a fuzzy closed set of X then (gof)(β) is a fuzzy mildly Minimal closed set in Z, Since (gof) is a fuzzy mildly Minimal-closed function. Since g is fuzzy mildly Minimal-irresolute function g^{-1} (gof)(β) is fuzzy mildly Minimal-closed in Y. But g^{-1} (gof)(β) = f(β) as g is injective. Thus f is fuzzy mildly Minimal-closed function.

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