IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

On New Generalization of Fuzzy Topological Spaces.

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ABSTRACT: In this paper,we introduce and investigate topological spaces called Fuzzy wgrα- Spaces and We get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.

Key words: Fuzzy wgrα-closed sets, Fuzzy wgrα-open sets, Fuzzy wgrα topological spaces.

AMS Mathematical Subject classification(2010):54C10,54C08, 54C05.

I. INTRODUCTION:

In the year 2015,R.S.Wali and Vivekananda Dembre introduced and studied Fuzzy wgrα-closed and open sets respectively.In this paper we define and study the properties of a new topological spaces called Fuzzy wgrα spaces.

II.PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c ,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1:A subset A of a topological space (X, τ) is called

(i) A pre generalized pre regular ωeakly closed set(briefly pgprω-closed set)[1] if pCl(A)⊆U whenever A⊆U and U is rgα open in (X,τ).

(ii)A subset A of a topological space (X,τ) is called pre generalized pre regular ωeakly open[2] (briefly pgprω-open) set in X if A^c is pgprω-closed in X.

(iii)Pre-open set [3] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

(iv)Regular generalized α-closed set(briefly,rgα-closed)[4]if αcl(A)⊆UwheneverA⊆Uand U is regularα-open in X.

(v) Generalized pre closed (briefly gp-closed) set [5] if pcl(A)⊆U whenever A⊆U and U is open in X.

(vi)Generalized pre regular closed set(briefly gpr-closed)[9] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

(vii)Definition [10]A topological space (X,T) is called pre-regular $T_{1/2}$-space if every gpr-closed set is preclosed.

Defintion 2.2: A map f:(X, τ) ⟶ (Y, σ)) is called

(i) [6]Fuzzy wgrα-continuous map if $f^{-1}(v)$ is Fuzzy wgrα closed in (X,τ) for every closed V in (Y, σ).

(ii) [7]Fuzzy wgrα-irresolute map if $f^{-1}(v)$ is Fuzzy wgrα closed in (X,τ) for every Fuzzy wgrα-closed V in (Y, σ).

(iii)[8] Fuzzy wgrα-closed map if $f^{-1}(v)$ is Fuzzy wgrα closed in (X,τ) for every closed V in (Y, σ).

(iv)[8]Fuzzy wgrα-open map if $f^{-1}(v)$ is Fuzzy wgrα closed in (X,τ) for every closed V in (Y, σ).

(v)[11]p-irresolute map if $f^{-1}(v)$ is p-closed in (X,τ) for every p-closed V in (Y, σ).

III On Fuzzy weakly Generalized regular AlphaTopological Spaces.

Definition 3.1: A topological space X is called a $\tau_{\text{Fuzzy wgr}\alpha}$ space if every Fuzzy wgrα-closed set in it is pre-closed.

Example 3.2:X={a,b,c,d},τ={X,Φ,{a},{b,{a,b},{a,b,c}}.Here every Fuzzy wgrα-closed set is pre Closed .So (X, τ) is a $\tau_{\text{Fuzzy wgr}\alpha}$– space.

Example 3.3:X={a,b,c,d}, τ={X,Φ,{a},{b},{a,b,{a,b,c}}.Here {a,d} is Fuzzy wgrα-closed,but not pre-closed So (X,T) is not $T_{\text{Fuzzy wgr}\alpha}$– space.

Theorem 3.4: A topological space X is a $\tau_{\text{Fuzzy wgr}\alpha}$ space iff for each x of X, {x} is either rgα -closed or pre-open.

Proof: Hypothesis: X is a $\tau_{\text{Fuzzy wgr}\alpha}$ -space.Let xϵX. If {x} is rgα -closed, then there is nothing to prove. If {x} is not rgα -closed, then X-{x} is not rgα -open and so X is the only rgα -open set containing X-{x} and pcl(X-{x})⊆X. Therefore X-{x} is Fuzzy wgrα-closed. X is $T_{\text{Fuzzy wgr}\alpha}$ – space (hypothesis) and X-{x} is Fuzzy wgrα-closed.Therefore X-{x} ispre-closed . Therefore {x} is pre-open. Thus for every x of X, a $\tau_{\text{Fuzzy wgr}\alpha}$ –space, {x} is either rgα -closed or pre-open.

Conversely, suppose for every x∊X, {x} is either rgα -closed or pre-open.Let A be a Fuzzy wgrα-closed subset of X. Now to prove A is pre-closed, we prove pcl(A) ⊆A.

Let x ϵ pcl(A).Then by hypothesis (a) {x} is pre-open ;If x is not in A , then A⊆{x }; a pre-closed setThereforepcl(A) ⊆{x}.'. x ϵ {x} which is not trueTherefore x ϵ A. Therefore pcl(A) ⊆A.

Thus every Fuzzy wgrα-closed set is pre-closed. Therefore X is a $\tau_{\text{Fuzzy wgr}\alpha}$ - space.

Theorem 3.5: Every pre-regular $T_{1/2}$-space is $\tau_{\text{Fuzzy wgr}\alpha}$ – space.

Proof: Let X be a pre-regular $\tau_{1/2}$-space and A be a Fuzzy wgrα-closed set. As every pgrgw-closed set is gpr-closed, A is gpr-closed. Since X is pre-regular $\tau_{1/2}$-space, A is preclosed. So every pgpgw-closed set is preclosed. Therefore X is $\tau_{\text{Fuzzy wgr}\alpha}$– space.

Converse of the above theorem is not true.

For example 3.6: X= {a,b,c,d}, τ={X,Φ, {a}, {b}, {a,b},{a,b,c}}. Here (X, T) is a $\tau_{\text{Fuzzy wgr}\alpha\text{-}}$ space,but not a pre-regular $\tau_{1/2}$-space.

Defintion 3.7: Let (X, τ) be topological space and τ-Fuzzy wgrα ={V⊆X : pgprω–cl(V^c) = V^c }, τ-pgprω is toplogy on X.

Theorem 3.8: Let f: X→Y be a function. Let (X,τ) and (Y,σ) be any two spaces such that $\tau_{pgpr\omega}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, f(Fuzzy wgrα-cl(A)) ⊆ cl(f(A)) holds,

(ii) f: (X, $\tau_{pgpr\omega}$)→(Y,σ) is continuous.

Proof: Suppose (i)holds.Let A be closed in Y.By hypothesf(Fuzzy wgrα-cl(f^{-1}(A)))⊆cl(A)=A.i.e. Fuzzy wgrα-cl(f^{-1}(A))⊆f^{-1}(A).Also f^{-1}(A)⊆pgprωcl(f^{-1}(A)).Hence Fuzzy wgrα-cl(f^{-1}(A))= f^{-1}(A). This implies f^{-1}(A)∈ $\tau_{pgpr\omega}$.Thus f^{-1}(A) is closed in (X, $\tau_{pgpr\omega}$) and so f is continuous.

This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: (X, $\tau_{pgpr\omega}$)→(Y,σ) is continuous, f^{-1}(cl(A)) is closed in (X, $\tau_{\text{Fuzzy wgr}\alpha}$) that implies by definition 3.7 Fuzzy wgrα-cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))). Now we have,A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) and by Fuzzy wgrα-closure, Pgprω-cl(A)⊆pgprω-cl(f^{-1}(cl(f(A))).Therefore f(Fuzzy wgrα-cl(A))⊆ cl(f(A)). This proves (i).

Theorem 3.9: Let X and Y be pgprω τ_p-spaces, then for a function f: (X,τ) → (Y,σ), the following are equivalent:

(i) f is p-irresolute map.

(ii) f is pgprω-irresolute map.

Proof: (i)=> (ii): Let f: (X,τ) →(Y,σ) be a p-irresolute map. Let V be a pgprω -closed set in Y. As Y is Fuzzy wgrαT_p-space, then V be a p-closed set in Y. Since f is p–irresolute map, f^{-1} (V) is p -closed in X. But every p-closed set is Fuzzy wgrα -closed in X and hence f^{-1} (V) is a pgprω-closed in X. Therefore, f is Fuzzy wgrα-irresolute map.

(ii)=> (i): Let f: (X,τ) → (Y,σ) be a pgprω –irresolute map. Let V be a p-closed set in Y. But every p-closed set is pgprω -closed set and hence V is pgprω-closed set in Y and f is Fuzzy wgrα-irresolute map implies f^{-1} (V) is Fuzzy wgrα-closed in X. But X is Fuzzy wgrαT_p-space and hence f^{-1} (V) is p-closed set in X. Thus, f is p-irresolute map.

Theorem 3.10: If a mapping f: (X, τ) → (Y, σ) is Fuzzy wgrα–closed map,then Fuzzy wgrα–cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof. Suppose that f is Fuzzy wgrα–closed and A ⊆X. Then cl(A) is closed in X and so f(cl(A)) is Fuzzy wgrα–closed in (Y, σ). We have f(A) ⊆ f(cl(A)), by Theorem 3.8 , Fuzzy wgrα–cl(f(A)) ⊆ Fuzzy wgrα–cl(f(cl(A))) → (i). Since f(cl(A)) is Fuzzy wgrα– closed in (Y,σ), Fuzzy wgrα–cl(f(cl(A))) = f(cl(A)) → (ii), by the Theorem 3.8. From (i) and (ii), we have Fuzzy wgrα–cl(f(A))⊆f(cl(A)) forevery subset A of (X, τ).

Corollary 3.11: If a mapping f: (X, τ) → (Y, σ) is a pgprω–closed, then the image f(A) of closed set A in (X, τ) is $\tau_{pgpr\omega}$–closed in (Y, σ).

Proof. Let A be a closed set in (X, τ). Since f is Fuzzy wgrα–closed, by above Theorem 3.10,Fuzzy wgrα–cl(f(A)⊆f(cl(A))→(i). Also cl(A)=A, as A is a closed set and so f(cl(A)) = f(A) →(ii). From (i) and (ii), we have Fuzzy wgrα–cl(f(A))⊆f(A). We know that f(A)⊆Fuzzy wgrα–cl(f(A)) and so Fuzzy wgrα–cl(f(A)) =f(A). Therefore f(A) is $\tau_{pgpr\omega}$–closed in (Y, σ).

Theorem 3.12: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is Fuzzy wgrα–closed maps and (Y, σ) be a $T_{pgpr\omega}$-space then g∘f: (X, τ) → (Z, η) is pgprω–closed map.

Proof. Let A be a closed set of (X, τ). Since f is pgprω–closed,f(A) is pgprω–closed in (Y, σ). Then by hypothesis, f(A) is closed. Since g is pgprω–closed, g(f(A)) is Fuzzy wgrα–closed in (Z, η) and g(f(A)) = g∘f(A). Therefore g∘f is pgprω–closed map.

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