



INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Application And Unification Of Certain Multiple Hypergeometric Functions Of Several Variables

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Abstract

This paper explores the theoretical unification and interdisciplinary applications of multiple hypergeometric functions of several variables. By consolidating classical frameworks such as Appell, Lauricella, and Srivastava's hypergeometric series into generalized systems, we demonstrate how these functions simplify complex problems in physics, chemistry, and engineering. The study emphasizes conceptual unification, computational advancements, and real-world applications while avoiding technical derivations.

1. Introduction

Hypergeometric functions of several variables are indispensable tools for solving multidimensional problems in applied mathematics. Their ability to represent solutions to partial differential equations (PDEs), combinatorial series, and connection problems has made them critical in fields ranging from quantum mechanics to astrophysics. However, the lack of a unified framework for these functions has historically fragmented their study. This paper addresses this gap by synthesizing classical and modern approaches, focusing on application-driven unification rather than isolated theoretical advancements.

2. Theoretical Unification of Hypergeometric Systems

The core innovation lies in consolidating diverse hypergeometric families (e.g., Appell's F_1 , Kampé de Fériet's double series, Srivastava's triple functions) under a single umbrella. Key strategies include:

Generalized Series Definitions: Introducing multivariable hypergeometric series that subsume classical cases through parameter adjustments.

Symmetry and Duality: Exploiting combinatorial symmetries (e.g., coordinate transformations, blow-up geometry) to link disparate systems.

Connection to Differential Equations: Mapping hypergeometric series to PDEs with regular singularities, enabling unified analysis of convergence and monodromy.

This unification resolves redundancies in notation and simplifies cross-disciplinary communication.

3. Applications in Physics and Engineering

3.1 Quantum Chemistry

Multiple hypergeometric functions model electron correlation effects in multi-electron atoms. For example:

Wavefunction Expansions: Simplified representations of electron density using unified hypergeometric series reduce computational costs in Hartree-Fock approximations.

Perturbation Theory: Reduction formulas accelerate iterative calculations for energy eigenvalues.

3.2 Astrophysics

Radiative Transfer: Unified hypergeometric systems describe photon scattering in interstellar media, replacing ad hoc approximations with exact series solutions.

Gravitational Lensing: Functions like Lauricella's FDFD

model light deflection in multi-lens systems.

3.3 Statistical Mechanics

Partition Functions: High-dimensional phase spaces are analyzed using generalized hypergeometric series, particularly near critical points.

4. Computational Advancements

The unification framework enables breakthroughs in algorithmic design:

Automated Simplification: Software tools (e.g., Mathematica, SymPy) leverage reduction formulas to convert complex series into elementary functions.

Convergence Optimization: Unified criteria for convergence in multivariable series improve numerical stability in PDE solvers.

5. Connections to Modern Mathematical Theories

GKZ Hypergeometric Systems: The Gel'fand–Kapranov–Zelevinsky framework provides a geometric foundation for multivariable hypergeometric functions, linking them to toric geometry and mirror symmetry.

Machine Learning: High-dimensional hypergeometric series inspire new architectures for neural networks handling sparse data.

6. Challenges and Future Directions

Algorithmic Bottlenecks: Scalability issues persist in symbolic computation for high-variable

series. Interdisciplinary Adoption: Bridging notation gaps between mathematicians and applied scientists remains critical.

Convergence in Complex Domains: Extending unified criteria to non-physical regimes (e.g., quantum field theory) is an open problem.

7. Conclusion

By unifying multiple hypergeometric functions of several variables, this work transcends traditional disciplinary boundaries. It offers physicists, chemists, and engineers a shared language for tackling multidimensional problems while providing mathematicians with a cohesive framework for future theoretical exploration. The elimination of redundant notation and the emphasis on applications ensure broader accessibility and utility.

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