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Employing Sugeno's Technique To Evaluate The Compressor House Unit (CHU) System's Vague Reliability

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Abstract: The vague reliability of the system is studied usually by using classical set theory. Nevertheless, analyzing the system's actual state cannot be done using classical set theory. Given that today's current techniques are the outcome of continuous advancements in science and engineering technology, fuzzy set theory is one of the best theories to account for subjectivity and vagueness in information. This article aims to analyze mechanical systems vague reliability, specifically as it relates to the compressor house unit (CHU). Four subsystems make up the compressor house unit. The compressor house unit's vague reliability is being assessed using the parallel series network method, which is determined by combining Sugano's fuzzy failure rate methodologies with trapezoidal vague numbers (TrapVan). In this article, the authors also applied the suggested method for estimating fuzzy reliability by Sugeno's fuzzy method with the help of parallel-series and series-parallel system methods for TrapVaN. The approach using a mathematical illustration and a graphical representation has been performed in this article.

***Index Terms* - Vague Sets (VaS), Compressor House Unit (CHU), Fuzzy Failure Rate (FFR), Trapezoidal Vague Numbers (TrapVaN)**

1. INTRODUCTION

The definition of the fuzzy set and the value of the membership grade lying in the interval [0,1] were introduced by [23]. The notions of inclusion, union, intersection, complement, relation, convexity, etc. for these fuzzy sets are also given by Zadeh. Numerous real-world applications, such as sensor data, expert systems, decision analysis, medical sciences, management, and engineering challenges, include vaguely specified data values. [1] explored the many aspects of fuzzy logic and proposed the concept of fuzzy approaches for detecting system crashes. A communication network is one of the world's most advanced system products. Network systems employ many components to integrate more functionality under system command and control. In addition to analyzing HFS and DHFS mathematical examples, [2] also computed OFC system's the fuzzy reliability using HFS and DHFS. [4] presented measures of similarity between vague sets. [3]-[3] presented fuzzy set theory-based fuzzy system reliability analysis, in which vague sets represent the reliability of the system component in the [0,1] discourse universe. [6]-[6] supported the use of fuzzy numbers in the confidence of possibility and the possibility of taking into account factors to assess the fuzzy reliability's timeline and limit quantity.
[16] introduced the notions of mathematical action on a fuzzy set and a vague set-theoretic technique for fault tree research. In 2010, [16] also presented a VaS theoretic approach to fault tree analysis and the notion of mathematical rules on vague sets, where both membership and non-membership are used to convey the salient features of the system. According to [14], fuzzy processes, which resemble stochastic processes, were used to characterize the reliability of systems. They also attempted to investigate fuzzy tools about series, parallel, bridge, and k-out of n communication networks. According to [8], a group of items is called

a fuzzy set if each object's membership degree falls on a continuous subinterval of [0,1]. Its distinctive characteristics are its working and non-working membership functions.
[8] reports the reliability study of a gas separation system using certain variable components. The Boolean function approach is used to analyze that model, taking into account the Weibull distribution of the system's constituent parts. Generalized Trapezoidal Fuzzy Numbers (GTrFN) were proposed by [9] as a means of evaluating the fuzzy reliability of the structure. [11] presented a novel method for executing different arithmetic operations among different kinds of fuzzy sets. The triangular intuitionistic fuzzy number is used to evaluate the fuzzy reliability of the system, which is a way to represent the reliability of every system's component. [11] introduces the arithmetic operations of triangular intuitionistic fuzzy numbers. The fuzzy set theory was established by [13] to replace the standard probability distributions used for the system's pieces in order to alleviate the limitations of classical possibilities in accounting for fundamental uncertainty in possible data. To ascertain the fuzzy reliability of the system, they compute the value of the fuzzy number using non-linear programming techniques. [14] calculated the reliability indices of the CHU system, including availability, failure rate, MTBF, repair time, ENOF, and reliability, at various uncertainty levels for a coal-fired thermal power plant.

[19] looked at attempts to assess fuzzy tools for network system reliability issues. They examined the system's vague reliability and failure rates using Sugeno's fuzzy failure rate calculation. With the use of two algorithms, [18] proposed an approach based on vague set theory to investigate the vague reliability of peel remover plants. [17] explained a generic three-level Takagi Sugeno extended fuzzy network and suggested a general multi-level network architecture formed using Takagi Sugeno (TSK) fuzzy inference systems as essential components. Based on the Sugeno [TSK] fuzzy model, [19] established a novel technique for estimating system parameters. Using Sugeno's fuzzy model, they examined the fuzzy reliability of a three-unit system. The analytical structure studied by [23] is related to a wide class of fuzzy systems developed by Takagi-Sugeno that employ arbitrary fuzzy rules. Recently, we proposed simpler linear rule consequences for some well-known models, filters, and non-fuzzy controllers.

2.System Descriptions

A medium-sized coal-fired thermal power plant (TPP) situated in India with a 1368 MW capacity has been considered. It is composed of several functional subsystems, including a coal crushing unit (CCU), coal handling plant (CHP), steam generating unit (SGU), water treatment plant (WTP), compressor house unit (CHU), power generating unit (PGU), ash handling unit (AHU), etc. In this article, we will talk about the CHU system, which is one of the main working elements of a TPP. The air drier system (ADS), air cleaner (AC), air storage tank (AST), and compressed air system (CAS) comprise the CHU system. After entering the CAS, the ambient air is compressed. Compressed air is kept in an air tank and then goes through a dryer and heater to remove moisture. After going through the cleaner, the dry air is sent to various plant subsystems so that their valves can operate properly [13]. Figure 1 displays the CHU diagram. The following four primary subsystems make up the CHU system:

I. **CAS**: This system, which is made up of the side compressor at low pressure, an intercooler, and the side compressor at high pressure, is connected in a line. Compressed air at both high and low pressure is produced by this method.
II. **AST**: This system, which pressurizes air, is kept there for later use.
III. **ADS**: This system is used to remove humidity from compressed air and consists of a dryer for air and an air heater that are connected in a line.
IV. **AC**: Dry air is cleaned with this technique in preparation for usage.

All four of the above subsystems are linked together by a series.

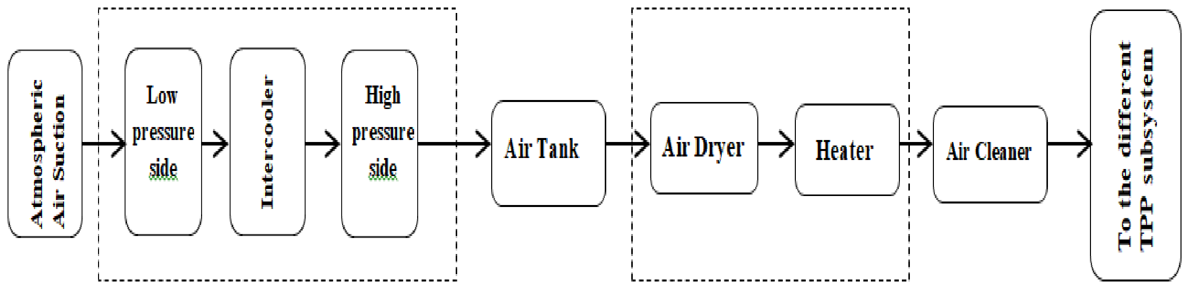

Figure 1: Compressor House Unit (CHU) System

3. SEVERAL FUNDAMENTAL DEFINITIONS

Vague sets have related definitions.

1.1. Vague Set (VaS): A set $\tilde{S}$ of the universal set U described as a vague set if $\tilde{S}$ has the following form, as shown in equation (1):

$$\tilde{S} = \{z \in U :< z, \mu_{\tilde{s}}(z), \nu_{\tilde{s}}(z) >\} \qquad (1)$$

In the equation (1) the degree of membership $\mu_{\tilde{s}}(z)$ is a function from $U \to [0,1]$ and the degree of non-membership $\nu_{\tilde{s}}(z)$ is a function from $U \to [0,1]$ of the component $z \in U$ to S respectively, which satisfies the condition $0 \le \mu_{\tilde{s}}(z) + \nu_{\tilde{s}}(z) \le 1 \;\; \forall z \in U$.

If there is a finite number of a set U in the universal set then the equation (1) is possible to write as

$$\tilde{S} = \sum_{m=1}^{n} \frac{[\mu_{\tilde{s}}(z), 1-\nu_{\tilde{s}}(z)]dz}{z_m} \quad \forall z_m \in U$$

If there is a finite number of a set U in the universal set then the equation (1) can be written as

$$\tilde{S} = \int_E \frac{[\mu_{\tilde{s}}(z), 1-\nu_{\tilde{s}}(z)]dz}{z_m} \quad \forall z_m \in U$$

The graphically VaS can be indicated as (Figure 2)

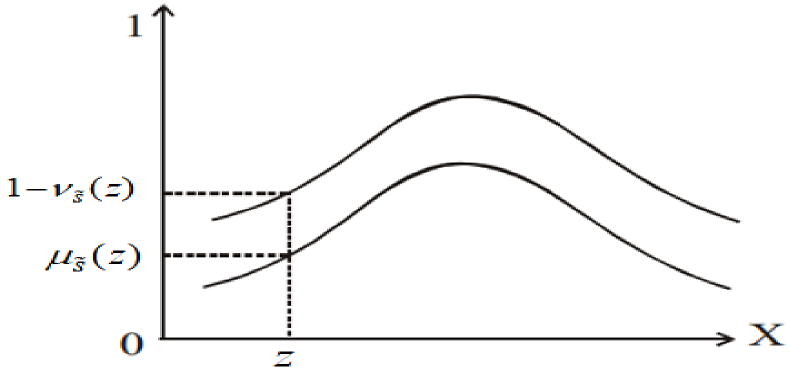

Figure 2: Membership and non-membership functions of $\tilde{S}$

1.2. $(\alpha,\beta)-$cuts for Vague Set

If $\tilde{S}$ is a vague set then, a set of $(\alpha,\beta)-$cut generated by $\tilde{S}$ is indicated by $\tilde{S}_{\alpha,\beta}$ and described as:

$$\tilde{S}_{\alpha,\beta} = \{z \in U :< z, \mu_{\tilde{s}}(z), \nu_{\tilde{s}}(z) >, \alpha \le \mu_{\tilde{s}}(z), \nu_{\tilde{s}}(z) \le \beta\} \qquad (2)$$

$\forall\, \alpha,\beta \in [0,1]$ with the condition $\alpha+\beta \le 1$. In equation (2) $(\alpha,\beta)-$cut for vague set $\tilde{S}_{\alpha,\beta}$ is the crisp set of components $z \in \tilde{S}$ maximally to the degree β and at lowest to the degree α, which relates to $\tilde{S}$.

3.3. Vague Number (VaN)

A vague number of the vague set $\tilde{S}$ is specified as:

- **(a)** A vague subset of real numbers R
- **(b)** The vague set $\tilde{S}$ is normal if $\exists\, z_0 \in R$ such that $1-\nu_{\tilde{s}}(z_0)=1$ i.e. $\nu_{\tilde{s}}(z_0)=0$
- **(c)** The vague set $\tilde{S}$ is convex for the membership $\mu_{\tilde{s}}(z)$ and non-membership $\nu_{\tilde{s}}(z)$ then

$\mu_{\tilde{s}}[\lambda_1 z_1 + \lambda_2 z_2] \ge \min[\mu_{\tilde{s}}(z_1), \mu_{\tilde{s}}(z_2)]$ for every $z_1, z_2 \in R$ and $\lambda_1, \lambda_2 \in [0,1]$ with $\lambda_1 + \lambda_2 = 1$

$1-\nu_{\tilde{s}}[\lambda_1 z_1 + \lambda_2 z_2] \ge \min[\{1-\nu_{\tilde{s}}(z_1)\}, \{1-\nu_{\tilde{s}}(z_2)\}]$ for every $z_1, z_2 \in R$ and $\lambda_1, \lambda_2 \in [0,1]$ with $\lambda_1 + \lambda_2 = 1$

- **(d)** Concave for $\mu_{\tilde{s}}(z)$ and $\nu_{\tilde{s}}(z)$

$1-\mu_{\tilde{s}}[\lambda_1 z_1 + \lambda_2 z_2] \ge \max[\{1-\mu_{\tilde{s}}(z_1)\}, \{1-\mu_{\tilde{s}}(z_2)\}]$ for every $z_1, z_2 \in R$ and $\lambda_1, \lambda_2 \in [0,1]$ with $\lambda_1 + \lambda_2 = 1$

$\nu_{\tilde{s}}[\lambda_1 z_1 + \lambda_2 z_2] \ge \max[\nu_{\tilde{s}}(z_1), \nu_{\tilde{s}}(z_2)]$ for every $z_1, z_2 \in R$ and $\lambda_1, \lambda_2 \in [0,1]$ with $\lambda_1 + \lambda_2 = 1$

There are many forms of fuzzy numbers, but authors concentrate only trapezoidal fuzzy numbers and explore trapezoidal vague numbers with $(\alpha,\beta)-$cuts.

3.4. Trapezoidal vague numbers (TrapVaN)

In the real line R, a Trapezoidal vague number $\tilde{S}$ may be expressed as $(c_1, c_2, c_3, c_4; c_1', c_2', c_3', c_4')$, where $(c_1 \le c_2 \le c_3 \le c_4; c_1' \le c_2' \le c_3' \le c_4')$, (Figure 3), and having the following membership and non-membership functions shown in equation (3) and (4) as:

$$\mu_{\tilde{S}}(z) = \begin{cases} \dfrac{z - c_1}{c_2 - c_1} & \text{when } c_1 \le z \le c_2 \\ 1 & \text{when } c_2 \le z \le c_3 \\ \dfrac{c_4 - z}{c_4 - c_3} & \text{when } c_3 \le z \le c_4 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

And

$$\nu_{\tilde{S}}(z) = \begin{cases} \dfrac{c_2 - z}{c_2 - c_1'} & \text{when } c_1' \le z \le c_2 \\ 0 & \text{when } c_2 \le z \le c_3 \\ \dfrac{z - c_3}{c_4' - c_3} & \text{when } c_3 \le z \le c_4' \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

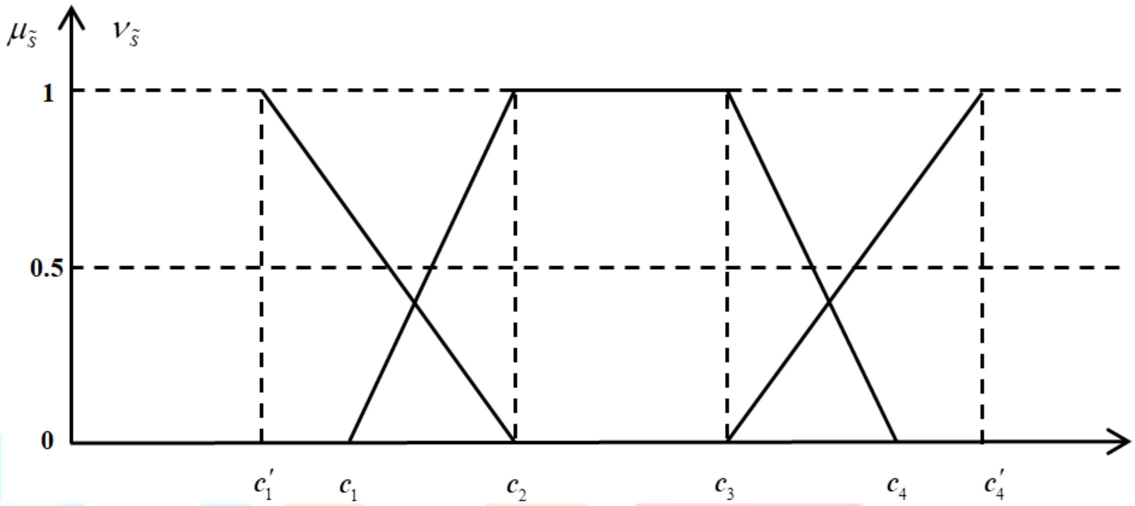

Figure 3: $\mu_{\tilde{S}}(y)$ and $\nu_{\tilde{S}}(y)$ of TrapVaN

3.5. Parallel and Series System for TrapVaN

Let's think about the parallel network depicted in Figure 4, which have n components. The vague reliability $\tilde{R}_P$ of the parallel network may be used by below equation, which is given below:

$\tilde{R}_P = 1!\ \{(1!\ \tilde{R}_1)(1!\ \tilde{R}_2).......(1!\ \tilde{R}_n)\}$

If $\tilde{R}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4}; r_{i1}', r_{i2}', r_{i3}', r_{i4}')$, then the vague reliability of the parallel network in TrapVaN may be depicted as:

$$\tilde{R}_P = \begin{bmatrix} \{(1-(1-r_{11})(1-r_{21})....(1-r_{n1})), (1-(1-r_{12})(1-r_{22})....(1-r_{n2})), (1-(1-r_{13})(1-r_{23})....(1-r_{33})), (1-(1-r_{14})(1-r_{24})....(1-r_{n4}))\}; \\ \{(1-(1-r_{11}')(1-r_{21}')....(1-r_{n1}')), (1-(1-r_{12}')(1-r_{22}')....(1-r_{n2}')), (1-(1-r_{13}')(1-r_{23}')....(1-r_{n3}')), (1-(1-r_{14}')(1-r_{24}')....(1-r_{n4}'))\} \end{bmatrix}$$

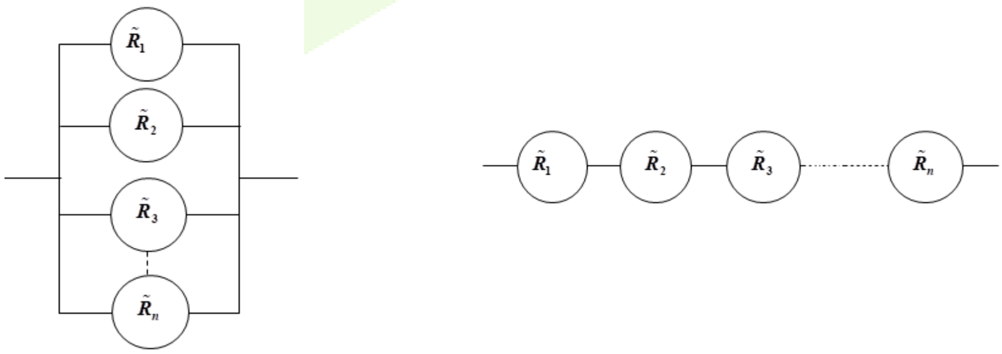

Figure 4: Parallel and Series System

The vague reliability of the series network $\tilde{R}_S$, which have n components as depicted in Figure 4, may be represented by the following equation $\tilde{R}_P = \tilde{R}_1 \otimes \tilde{R}_2 \otimes ..... \otimes \tilde{R}_n$.

If $\tilde{R}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4}; r_{i1}', r_{i2}', r_{i3}', r_{i4}')$, then the vague reliability of the series network in TrapVaN may be depicted as: $\tilde{R}_S = [\{r_{11}r_{21}....r_{n1}, r_{12}r_{22}....r_{n2}, r_{13}r_{23}....r_{n3}, r_{14}r_{24}....r_{n4}\}; \{r_{11}'r_{21}'....r_{n1}', r_{12}'r_{22}'....r_{n2}', r_{13}'r_{23}'....r_{n3}', r_{14}'r_{24}'....r_{n4}'\}]$

4. SUGENO'S TECHNIQUE

[21]-[21] proposed the Sugeno fuzzy model, which calls for using a singleton as the consequent membership degree of the rule. A fuzzy singleton is a fuzzy set whose degree of membership is 1 at one location in the discourse universe and 0 at all other locations. The membership degree of this technique will be either linear or constant. This strategy is in line with Mamdani's, however it uses fuzzy operations in the first two steps and fuzzifies the inputs. The outcome of Sugeno's method is either linear or constant.

4.1. Suggested Method for Estimating the FFR in the Fuzzy Model of Sugeno's

When determining the reliability features of any type of system, failure or repair rates are an important consideration. A small error in the failure rate could result in an overestimation or underestimation of the system's reliability; this risk needs to be minimized as much as possible for systems with humanitarian applications. Estimates and the highest probability approach are commonly applied to a variety of sets of data. During the process of gathering failure information, the following queries may come up:

(a) Failure occurs as planned, but it may go unnoticed or is not accurately documented when it occurs.
(b) There is either no failure or very little failure. As a consequence, filtered observations led to the indicated time for failure.
(c) It is necessary to collect several failure data values under the same operational conditions. Operating circumstances are frequently described in vague ways and are challenging to articulate precisely.
(d) It may include imprecise human judgment, assessment, and conclusions at some stages.

In the above-described situations, failure data may be handled using fuzzy techniques. We provide a method for estimating failure rate parameters based on Sugeno's fuzzy model, which makes use of defuzzification, fuzzy aggregation, and fuzzy numbers concepts. The condensing procedure the collection of hazy findings into a single assessment is called defuzzification.
The approach's perspective depends on the following fundamental ideas of the fuzzy model of Sugeno's:

i. In this step, the failure data set's weight will be established.
ii. In a later step, the output that was obtained from the different sets of data will be found.

According to the above **i** and **ii** steps, we can find the weighted average of all the rules by using equation (5): To determine the ultimate result, we'll employ

$$Final\ output = \frac{\sum_{i=1}^{n} W_i Z_i}{\sum_{i=1}^{n} W_i} \tag{5}$$

A suggested algorithm uses $(\alpha,\beta)-$cuts to conduct mathematical operations among VaN, taking into account the VaN, which represents the reliability of various components. Given a VaN $\tilde{S}$, the $(\alpha,\beta)-$cut may be obtained as:

$$\tilde{S}_{\alpha,\beta} = \begin{cases} [S_1(\alpha), S_2(\alpha)] & for\ the\ degree\ of\ acceptances\ \alpha \in [0,1] \\ [S_1'(\beta), S_2'(\beta)] & for\ the\ degree\ of\ rejection\ \beta \in [0,1] \end{cases}\ with\ the\ condition\ \alpha+\beta \le 1$$

Here $\frac{dS_2(\alpha)}{d\alpha} < 0 < \frac{dS_1(\alpha)}{d\alpha}$ $for\ all\ \alpha \in (0,1)\ and\ S_1(1) \le S_2(1)\ with\ \frac{dS_1'(\beta)}{d\beta} < 0 < \frac{dS_2'(\beta)}{d\beta}$ $for\ all\ \beta \in (0,1)\ and\ S_1'(0) \le S_2'(0).$

The $(\alpha,\beta)-$cut for $\tilde{S}$ may be represented as: $\tilde{S}_{\alpha,\beta} = \{[S_1(\alpha), S_2(\alpha)], [S_1'(\beta), S_2'(\beta)]\}\ \forall\, \alpha,\beta \in [0,1],\ \alpha+\beta \le 1.$

1st. VaN for each network unit must be determined in the stage 1st.
2nd. In this step, we determine the $(\alpha,\beta)-$cut for each VaN as calculated in step1st.
3rd. In the step 3rd, if $\tilde{A}_1 = \{x; x'\}$ and $\tilde{A}_2 = \{y; y'\}$ are two TrapVaN, where $x = (x_1, x_2, x_3, x_4)$, $x' = (x_1', x_2', x_3', x_4')$, $y = (y_1, y_2, y_3, y_4)$ and $y' = (y_1', y_2', y_3', y_4')$ then sum $\tilde{A}_1 \oplus \tilde{A}_2 = \{x_1+y_1, x_2+y_2, x_3+y_3, x_4+y_4; x_1'+y_1', x_2'+y_2', x_3'+y_3', x_4'+y_4'\}$ is also a TrapVaN. The $\alpha-$cut method and finding the membership function $\mu_{\tilde{C}_1}(z)$ of acceptance VaS may be done by transforming $z = x+y$. Assume $\Delta_{x_r} = x_r - x_{r-1}$ and $\Delta_{y_r} = y_r - y_{r-1}$. The $\alpha-$cut for $\mu_{\tilde{A}_1}(x)$ is $\{x_1+\alpha\Delta_{x_2}, x_4-\alpha\Delta_{x_4}\}$ for every $\alpha \in [0,1]$ i.e. $z_1 \in \{x_1+\alpha\Delta_{x_2}, x_4-\alpha\Delta_{x_4}\}$, and the $\alpha-$cut for $\mu_{\tilde{A}_2}(y)$ is $\{y_1+\alpha\Delta_{y_2}, y_4-\alpha\Delta_{y_4}\}$ for every $\alpha \in [0,1]$ i.e. $z_2 \in \{y_1+\alpha\Delta_{y_2}, y_4-\alpha\Delta_{y_4}\}$. Therefore $z \in [x_1+y_1+\alpha\{\Delta_{x_2}+\Delta_{y_2}\}, x_4+y_4-\alpha\{\Delta_{x_4}+\Delta_{y_4}\}]$. Hence $\mu_{\tilde{C}_1}(z)$ of the sum $\tilde{C}_1 = \tilde{A}_1 \oplus \tilde{A}_2$ is stated as:

$$\mu_{\tilde{C}_1}(z)=\begin{cases}\dfrac{z-x_1-y_1}{\Delta_{x_2}+\Delta_{y_2}} & when\ x_1+y_1\le z\le x_2+y_2\\ 1 & when\ x_2+y_2\le z\le x_3+y_3\\ \dfrac{x_4+y_4-z}{\Delta_{x_4}+\Delta_{y_4}} & when\ x_3+y_3\le z\le x_4+y_4\\ 0 & otherwise\end{cases} \tag{6}$$

The equation (6) is the formula for the membership function of the sum of two TrapVaN. Assume $\Delta_{x_1'}=x_2-x_1'$, $\Delta_{x_4'}=x_4'-x_3$, $\Delta_{y_1'}=y_2-y_1'$, and $\Delta_{y_4'}=y_4'-y_3$. The $\beta-$cut for $\nu_{\tilde{A}_1}(x)$ is $\{x_2-\beta\Delta_{x_1'},x_3+\beta\Delta_{x_4'}\}$ for every $\beta\in[0,1]$ i.e. $z_1\in\{x_2-\beta\Delta_{x_1'},x_3+\beta\Delta_{x_4'}\}$, and the $\beta-$cut for $\nu_{\tilde{A}_2}(y)$ is $\{y_2-\beta\Delta_{y_1'},y_3+\beta\Delta_{y_4'}\}$ for every $\beta\in[0,1]$ i.e. $z_2\in\{y_2-\beta\Delta_{y_1'},y_3+\beta\Delta_{y_4'}\}$. Therefore $z=(z_1+z_2)\in[x_2+y_2+\beta\{\Delta_{x_1'}+\Delta_{y_1'}\},x_3+y_3-\beta\{\Delta_{x_4'}+\Delta_{y_4'}\}]$. Hence $\mu_{\tilde{C}_1}(z)$ of the sum $\tilde{C}_1=\tilde{A}_1\oplus\tilde{A}_2$ is stated as:

$$\nu_{\tilde{C}_1}(z)=\begin{cases}\dfrac{x_2+y_2-z}{\Delta_{x_1'}+\Delta_{y_1'}} & when\ x_1'+y_1'\le z\le x_2+y_2\\ 0 & when\ x_2+y_2\le z\le x_3+y_3\\ \dfrac{z-a_3-y_3}{\Delta_{x_4'}+\Delta_{y_4'}} & when\ x_3+y_3\le z\le x_4'+y_4'\\ 1 & otherwise\end{cases} \tag{7}$$

The equation (7) is the formula for the non-membership function $\nu_{\tilde{C}_1}(z)$ of the sum of two TrapVaN.

4th. In the step 4th, if $\tilde{A}_1$ and $\tilde{A}_2$ are two TrapVaN, then $\tilde{A}_1\otimes\tilde{A}_2=\{x_1y_1,x_2y_2,x_3y_3,x_4y_4;x_1'y_1',x_2'y_2',x_3'y_3',x_4'y_4'\}$ is also a TrapVaN. The $\alpha-$cut method and the transformation $z=x+y$ can be used to find the membership function $\mu_{\tilde{C}_1}(z)$ of acceptance VaS. The $\alpha-$cut for the membership function $\mu_{\tilde{A}_1}(x)$ is $\{x_1+\alpha\Delta_{x_2},x_4-\alpha\Delta_{x_4}\}$ for every $\alpha\in[0,1]$ i.e. $z_1\in\{x_1+\alpha\Delta_{x_2},x_4-\alpha\Delta_{x_4}\}$, and the $\alpha-$cut for the membership function $\mu_{\tilde{A}_2}(y)$ is $\{y_1+\alpha\Delta_{y_2},y_4-\alpha\Delta_{y_4}\}$ for every $\alpha\in[0,1]$ i.e. $z_2\in\{y_1+\alpha\Delta_{y_2},y_4-\alpha\Delta_{y_4}\}$. Therefore $z=(z_1\times z_2)\in[\{x_1+\alpha\Delta_{x_2}\}\{y_1+\alpha\Delta_{y_2}\},\{x_4+\alpha\Delta_{x_4}\}\{y_4-\alpha\Delta_{y_4}\}]$. Hence $\mu_{\tilde{C}_2}(z)$ of the product $\tilde{C}_2=\tilde{A}_1\otimes\tilde{A}_2$ is stated as:

$$\mu_{\tilde{C}_2}(z)=\begin{cases}\dfrac{-B_1+\sqrt{B_1^{\,2}-4A_1(x_1y_1-z)}}{2A_1} & when\ x_1y_1\le z\le x_2y_2\\ 1 & when\ x_2y_2\le z\le x_3y_3\\ \dfrac{B_2+\sqrt{B_2^{\,2}-4A_2(x_4\,y_4-z)}}{2A_2} & when\ x_3y_3\le z\le x_4y_4\\ 0 & otherwise\end{cases} \tag{8}$$

Where $A_1=\Delta_{x_2}\Delta_{y_2},B_1=y_1\Delta_{x_2}+x_1\Delta_{y_2},A_2=\Delta_{x_4}\Delta_{y_4}$ and $B_2=-\{y_4\Delta_{x_4}+x_4\Delta_{y_4}\}$. The equation (8) is the formula for the membership function $\mu_{\tilde{C}_2}(z)$ of the product of two TrapVaN. The $\beta-$cut for $\nu_{\tilde{A}_1}(x)$ is $\{x_2-\beta\Delta_{x_1'},x_3+\beta\Delta_{x_4'}\}$ for every $\beta\in[0,1]$ i.e. $z_1\in\{x_2-\beta\Delta_{x_1'},x_3+\beta\Delta_{x_4'}\}$, and the $\beta-$cut for $\nu_{\tilde{A}_2}(y)$ is $\{y_2-\beta\Delta_{y_1'},y_3+\beta\Delta_{y_4'}\}$ for every $\beta\in[0,1]$ i.e. $z_2\in\{y_2-\beta\Delta_{y_1'},y_3+\beta\Delta_{y_4'}\}$. Therefore $z=(z_1\times z_2)\in[\{x_2-\beta\Delta_{x_1'}\}\{y_2-\beta\Delta_{y_1'}\},\{x_3+\beta\Delta_{x_4'}\}\{y_3-\beta\Delta_{y_4'}\}]$. Hence $\nu_{\tilde{C}_1}(z)$ of the product $\tilde{C}_2=\tilde{A}_1\otimes\tilde{A}_2$ is stated as:

$$\nu_{\tilde{C}_2}(z)=\begin{cases}1-\dfrac{-B_1'+\sqrt{B_1'^{\,2}-4A_1'(x_1'y_1'-z)}}{2A_1'} & when\ x_1'y_1'\le z\le x_2'y_2'\\ 0 & when\ x_2'y_2'\le z\le x_3'y_3'\\ 1-\dfrac{B_2'+\sqrt{B_2'^{\,2}-4A_2'(x_4'y_4'-z)}}{2A_2'} & when\ x_3'y_3'\le z\le x_4'y_4'\\ 1 & otherwise\end{cases} \tag{9}$$

Where $A_1' = \Delta_{x_1'}\Delta_{y_1'}$, $B_1' = y_1'\Delta_{x_1'} + x_1'\Delta_{y_1'}$, $A_2' = \Delta_{x_4'}\Delta_{y_4'}$ and $B_2' = -\{y_4'\Delta_{x_4'} + x_4'\Delta_{y_4'}\}$. The equation (9) is the formula for the non-membership function $\nu_{\tilde{C}_2}(z)$ of the product of two TrapVaN.

5th. The defuzzified data will be obtained by applying operations from 3rd and 4th, which will be used to evaluate the overall framework's reliability.

The following procedures will be used to develop an alternative evaluation procedure for the vague failure rate:

(i) The process to be followed is first used to compute the failure rate. Before multiple numbers can be used to determine the failure rate, the process must be repeated multiple times, with steps 3rd and 4th controlling the $\mu_{\tilde{C}_1}(z)$, $\mu_{\tilde{C}_2}(z)$, and $\nu_{\tilde{C}_1}(z)$, $\nu_{\tilde{C}_2}(z)$, respectively.

(ii) The numbers obtained in **(i)** will be categorized as membership or non-membership using the fuzzification method.

(iii) At this point, we will acquire a single vague number by using the aggregation processes.

(iv) We will now apply the defuzzification process to membership and non-membership functions in order to generate a single crisp number.

(v) We will also provide the input weights and fuzzified $\mu_w(x)$ and $\nu_w(x)$ using **(i)**, where w is the weight function.

(vi) After the numbers from **(v)** are fed into the fuzzy intersection of $\mu_w(x)$ and $\nu_w(x)$ for the weights function in the fuzzy procedures (AND = min), the only fuzzy integer for $\mu_w(x)$ and $\nu_w(x)$, w is calculated.

(vii) Next, we shall defuzzify $\mu_w(x)$ and $\nu_w(x)$, to yield a unique result in the interval form.

According to the above steps, we can find the weighted average of all the rules by using equation (10): To determine the ultimate result, we'll employ

$$Final\ output = \frac{\sum_{i=1}^{n}(w_{i_1} + w_{i_2})(ax_i + bx_i + d)}{\sum_{i=1}^{n}(w_{i_1} + w_{i_2})} \quad (10)$$

This will provide us with a set of crisp numbers that we can use to calculate the failure rate.

5. Assumptions

Let

$x_1, x_2, and\ x_3$	represent the CAS subsystem state.
x_4	represent the AST subsystem state.
$x_5\ and\ x_6$	represent the ADS subsystem state.
x_7	represent the AC subsystem state.
$\tilde{R}_i\ (i = 1,2,3,4)$	represent the system's vague reliability of i^{th} unit states.
$\tilde{R}_{CAS}$	shows the vague reliability of sub-system CAS.
$\tilde{R}_{AST}$	shows the vague reliability of sub-system AST.
$\tilde{R}_{ADS}$	shows the vague reliability of sub-system ADS.
$\tilde{R}_{AC}$	shows the vague reliability of sub-system AC.

and $[\{(r_{i1}, r_{i2}, r_{i3}, r_{i4}), \mu_{ij}\}; \{(r'_{i1}, r_{i2}, r'_{i3}, r'_{i4}), \nu_{ij}\}\ for\ i = 1,2,....4\ and\ j = 1,2,....7]$ shows that all the imprecise vague reliability of sub-system $\tilde{R}_j$.

6. Block Diagram

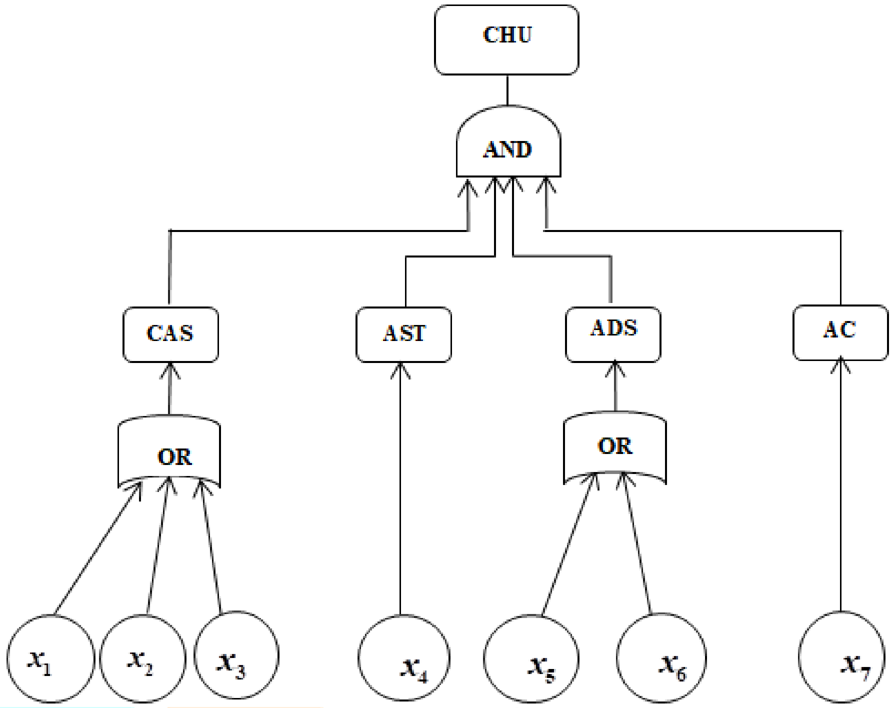

Figure 5: Fault Tree Diagram of Compressor House Unit (CHU) System

7. Vague Reliability Formulation of CHU System

Suppose we have to examine the vague reliability $\tilde{R}$ of the CHU system, as shown in Figure 5, with the help of the parallel and series network method. $\tilde{R}_{CAS}$ shows the vague reliability of the subsystem CAS and its describe as: $\tilde{R}_{CAS} = 1!\ (1!\ \tilde{R}_1)(1!\ \tilde{R}_2)(1!\ \tilde{R}_3)$

$$\tilde{R}_{CAS} = \begin{bmatrix} \{(1-(1-r_{11})(1-r_{21})(1-r_{31})),(1-(1-r_{12})(1-r_{22})(1-r_{32})),(1-(1-r_{13})(1-r_{23})(1-r_{33})),(1-(1-r_{14})(1-r_{24})(1-r_{34}))\min(\mu_{ij},\mu_{ji})\}; \\ \{(1-(1-r'_{11})(1-r'_{21})(1-r'_{31})),(1-(1-r'_{12})(1-r'_{22})(1-r'_{32})),(1-(1-r'_{13})(1-r'_{23})(1-r'_{33})),(1-(1-r'_{14})(1-r'_{24})(1-r'_{34}))\min(\nu_{ij},\nu_{ji})\} \end{bmatrix} \quad (11)$$

Equation (11) is the form of the TrapVaN. $\tilde{R}_{AST}$ shows the vague reliability of the subsystem AST and its describe as:

$$\tilde{R}_{AST} = \tilde{R}_4 = \left[\{(r_{41},r_{42},r_{43},r_{44});\min(\mu_{ij},\mu_{ji})\}\{(r'_{41},r'_{42},r'_{43},r'_{44});\min(\nu_{ij},\nu_{ji})\}\right] \quad (12)$$

Equation (12) is also the form of the TrapVaN. $\tilde{R}_{ADS}$ shows the vague reliability of the subsystem ADS and its describe as: $\tilde{R}_{ADS} = 1!\ (1!\ \tilde{R}_5)(1!\ \tilde{R}_6)$

$$\tilde{R}_{ADS} = \begin{bmatrix} \{(1-(1-r_{51})(1-r_{61})),(1-(1-r_{52})(1-r_{62})),(1-(1-r_{53})(1-r_{63})),(1-(1-r_{54})(1-r_{64}))\min(\mu_{ij},\mu_{ji})\}; \\ \{(1-(1-r'_{51})(1-r'_{61})),(1-(1-r'_{52})(1-r'_{62})),(1-(1-r'_{53})(1-r'_{63})),(1-(1-r'_{54})(1-r'_{64}))\min(\nu_{ij},\nu_{ji})\} \end{bmatrix} \quad (13)$$

Equation (13) represents TrapVaN form. $\tilde{R}_{AC}$ shows the vague reliability of the subsystem AC and its describe as:

$$\tilde{R}_{AC} = \tilde{R}_7 = \left[\{(r_{71},r_{72},r_{73},r_{74});\min(\mu_{ij},\mu_{ji})\}\{(r'_{71},r'_{72},r'_{73},r'_{74});\min(\nu_{ij},\nu_{ji})\}\right] \quad (14)$$

Equation (14) also represents TrapVaN form. As a result, the overall CHU system's vague reliability function $\tilde{R}$ is identified as follows:

$$\tilde{R} = \tilde{R}_{CAS} \otimes \tilde{R}_{AST} \otimes \tilde{R}_{ADS} \otimes \tilde{R}_{AC}$$

$$\tilde{R} = \{1!\ (1!\ \tilde{R}_1)(1!\ \tilde{R}_2)(1!\ \tilde{R}_3)\} \otimes \tilde{R}_4 \otimes \{1!\ (1!\ \tilde{R}_5)(1!\ \tilde{R}_6)\} \otimes \tilde{R}_7$$

$$\tilde{R} = \tilde{R}_4 \otimes \tilde{R}_7 \{(1!\ (1!\ \tilde{R}_1)(1!\ \tilde{R}_2)(1!\ \tilde{R}_3))(1!\ (1!\ \tilde{R}_5)(1!\ \tilde{R}_6))\}$$

The TrapVaN form of the above is shown in equation (15) as:

$$\tilde{R}=\left[\begin{array}{l}\left\{\left(\begin{array}{l}r_{41}r_{71}\left(1-(1-r_{11})(1-r_{21})(1-r_{31})\right)\left(1-(1-r_{51})(1-r_{61})\right),r_{42}r_{72}\left(1-(1-r_{12})(1-r_{22})(1-r_{32})\right)\left(1-(1-r_{52})(1-r_{62})\right),\\ r_{43}r_{73}\left(1-(1-r_{13})(1-r_{23})(1-r_{33})\right)\left(1-(1-r_{53})(1-r_{63})\right),r_{44}r_{74}\left(1-(1-r_{14})(1-r_{24})(1-r_{34})\right)\left(1-(1-r_{54})(1-r_{64})\right)\end{array}\right);\min\left(\mu_{ij},\mu_{ji}\right)\right\}\\ \left\{\left(\begin{array}{l}r'_{41}r_{71}\left(1-(1-r'_{11})(1-r'_{21})(1-r'_{31})\right)\left(1-(1-r'_{51})(1-r'_{61})\right),r'_{42}r'_{72}\left(1-(1-r'_{12})(1-r'_{22})(1-r'_{32})\right)\left(1-(1-r'_{52})(1-r'_{62})\right),\\ r'_{43}r'_{73}\left(1-(1-r'_{13})(1-r'_{23})(1-r'_{33})\right)\left(1-(1-r'_{53})(1-r'_{63})\right),r'_{44}r'_{74}\left(1-(1-r'_{14})(1-r'_{24})(1-r'_{34})\right)\left(1-(1-r'_{54})(1-r'_{64})\right)\end{array}\right);\min\left(\nu_{ij},\nu_{ji}\right)\right\}\end{array}\right] \quad (15)$$

7.1. Solution of Mathematical Problem:

Suppose the following are the vague reliability

$\tilde{R}_1=[(0.6828,0.7425,0.8590,0.9487);\ 0.7],[(0.6502,0.8339,0.8945,0.9094);0.8]$

$\tilde{R}_2=[(0.6427,0.7135,0.8073,0.9199);\ 0.7],[(0.6563,0.7222,0.7775,0.8057);0.8]$

$\tilde{R}_3=[(0.6795,0.7284,0.8567,0.9366);\ 0.7],[(0.6659,0.7279,0.7995,0.8156);0.8]$

$\tilde{R}_4=[(0.6774,0.7265,0.8556,0.9309);\ 0.7],[(0.6841,0.7374,0.7986,0.8239);0.8]$

$\tilde{R}_5=[(0.6911,0.7617,0.8757,0.9654);\ 0.7],[(0.6912,0.8933,0.9125,0.9298);0.8]$

$\tilde{R}_6=[(0.6452,0.7169,0.8165,0.9226);\ 0.7],[(0.6959,0.7671,0.8171,0.8365);0.8]$

$\tilde{R}_7=[(0.6702,0.7259,0.8540,0.9305);\ 0.7],[(0.6515,0.8999,0.9239,0.9381);0.8]$

After calculating the value of $\tilde{R}_{CAS},\tilde{R}_{AST},\tilde{R}_{ADS},and\ \tilde{R}_{AC}$ we get following values:

$\tilde{R}_{CAS}=[(0.9445,0.9800,0.9961,0.9997);\ 0.7],[(0.1274,0.4412,0.5505,0.5982);0.8]$

$\tilde{R}_{AST}=[(0.6374,0.7265,0.8556,0.9309);\ 0.7],[(0.8129,0.8713,0.9196,0.9363);0.8]$

$\tilde{R}_{ADS}=[(0.8904,0.9325,0.9772,0.9973);\ 0.7],[(0.4584,0.5938,0.6834,0.7125);0.8]$

$\tilde{R}_{AC}=[(0.6702,0.7259,0.8540,0.9305);\ 0.7],[(0.8306,0.8556,0.9331,0.9673);0.8]$

If $\tilde{R}$ is the system's vague reliability, using equation (15) and values already computed above, we get the required whole system's vague reliability $\tilde{R}$ as follows in equation (16):

$$\tilde{R}=[(0.3593,0.4819,0.7112,0.8636);\ 0.7],[(0.3310,0.5069,0.6906,0.8790);0.8] \quad (16)$$

8. Conclusion

A method based on vague set theory has been used to study the vague reliability $\tilde{R}$ of CHU system. Two algorithms have been used in this paper. The first is to evaluate the fuzzy failure rate of CHU system using Sugeno's technique, and the second is a definition of $(\alpha,\beta)-$cut set to evaluate vague numbers. In this work, the authors attempt to devise a method for estimating the CHU system's vague reliability value, $\tilde{R},$ and TrapVaN, which makes up the components of the CHU network system, provides the basis for this system's VaS theory. These TrapVaN are assessed for $(\alpha,\beta)-$cut. The CHU network systems that are series, parallel, or series-parallel are subjected to vague reliability $\tilde{R}$ analyses using mathematical operations over these assessed TrapVaN using (α,β)-cut. Equation (16) represents the data after the CHU system's vague reliability $\tilde{R}$ has been calculated. Figure 6 is based on the data from equation (16), and it shows that the data is discovered to be in the form of a trapezoidal shape.

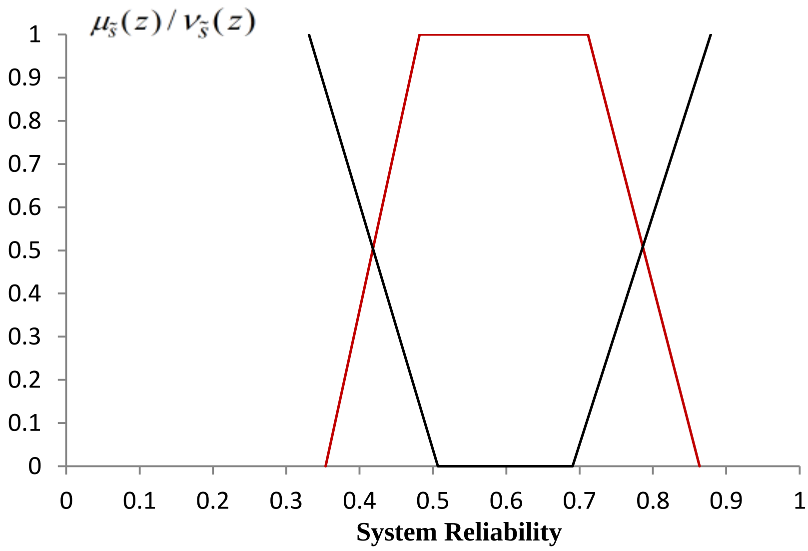

Figure 6: A graphic depiction of the CHU system's vague reliability

ACKNOWLEDGMENT

Entitled "Employing Sugeno's Technique to Evaluate the Compressor House Unit (CHU) System's Vague Reliability" is the authentic research paper we would like to submit for thought in the "International Journal of Fuzzy Systems." On behalf of all the writers, I, Mahesh Chand, certify that this work is unique, unpublished, and not presently being considered for publication anywhere. In this research, we report on a compressor house unit (CHU) system and attempt to assess its trustworthiness in the public domain under certain particular scenarios by applying Sugeno's fuzzy failure rate techniques to calculate the fuzzy reliability.

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