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Midly Properties in Topological Spaces

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Abstract: In this paper, the notation of minimal midly Generalized interior and minimal midly Generalized closure are introduced and studied in topological spaces. It is proved that the complment of minimal midly Generalized-interior of A is the closure of the complement of A and some properties of the new concepts have been studied.

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I. Introduction

N.Levine[1] introduced the concept of generalized sets of a topological space in 1970.Dunham[2] defined generalized closure operator in 1982.Mashhour,Abd El-Monsef and Deeb[3] introduced the concept of pre-closed sets in 1982.Gnanambal[4] introduced and studied the concept of gpr closed sets in topological space. Gnanambal and Balachandran[5], introduced and studied the concept ofgpr- interior and gpr-closure operator in topological space in 1999. Recently In the year 1999 N.Nagaveni [6] introduced and studied semi weakly generalized closed sets (briefly-minimal midly Generalized)and semi weakly generalized open [7] sets in topological spaces.

2. PRELIMINARIES

A subset A of a topological space (X,τ) is called

(i) Generalized closed set(briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(ii) Pre-openset if A $\subseteq$int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.

(iii) Generalized pre regular closed set(briefly gpr-closed) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.

(iv) A subset A of topological space (X,τ) is called a pre generalized pre regular ωeakly closed set(briefly pgprω-closed set) if $pCl(A)\subseteq U$ whenever $A\subseteq U$ and U is rgα open in (X,τ).

(v) A subset A in (X,τ) is called semi weakly generalized open set in X if A^c is semi weakly generalized closed set in X.

(vi) Mildly generalized closed (briefly mildly g-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Throughout this paper space $(X, \tau)$and(Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A,pre closure of A and pre-interior of A in X respectively.

3.minimal midly Generalized-closure and minimal midly Generalized- Interior

In this section the notation of minimal midly Generalized closure and minimal midly Generalized Interior is defined and some of its basic properties are studied.

Definition 3.1 : For a subset A of (X,τ), minimal midly Generalized-closure of A is denoted by minimal midly Generalized-cl(A) and defined as minimal midly Generalized-cl(A)= $\cap\{G:A\subseteq G$, G is minimal midly Generalized–closed in $(X,\tau)\}$ or $\cap\{G:A\subseteq G, G\epsilon$minimal midly Generalized-C(X) }

Theorem3.2: If A and B are subsets of space (X,τ) then

(i) minimal midly Generalized cl (X) = X ,minimal midly Generalized cl(ϕ)=ϕ

(ii) $A \subseteq$ midly Generalized -cl(A)

(iii)If B is any minimal midly Generalized-closed set containing A, then minimal midly Generalized cl(A)$\subseteq$ B.

(iv)If $A\subseteq B$ then minimal midly Generalized cl(A) $\subseteq$ midly Generalized cl(B)

(v) minimal midly Generalized-cl(A) = minimal midly Generalized-cl(minimal midly Generalized-cl(A))

(vi) minimal midly Generalized-cl(A∪B) = minimal midly Generalized-cl(A)∪minimal midly Generalized-cl(B).

Proof:

(i) By definition of minimal midly Generalized-closure, X is only minimal midly Generalized closed set containing X. Therefore minimal midly Generalized-cl(X) = Intersection of all the minimal midly Generalized-closed set containing X = ∩{X} =X .Therefore minimal midly Generalized-cl (X) = X and again by definition of minimal midly Generalized-closure.

minimal midly Generalized-cl(ϕ) = Intersection of all minimal midly Generalized-closed sets containing ϕ.

= ϕ∩any minimal midly Generalized-closed set containing ϕ= ϕ.

Therefore minimal midly Generalized-cl(ϕ)=ϕ.

(ii) By definition of minimal midly Generalized-closure of A, it is obvious that A ⊆ minimal midly Generalized-cl(A).

(iii)Let B be any minimal midly Generalized-closed set containing A. Since minimal midly Generalized-cl(A) is the intersection of all minimal midly Generalized-closed set containing A, minimal midly Generalized-cl(A) is contained in every minimal midly Generalized-closed set containing A. Hence in particular minimal midly Generalized-cl(A)⊆B.

(iv) Let A and B be subsets of (X,τ) such that A⊆B by definition minimal midly Generalized-closure , Minimal midly Generalized-cl(B)=∩{F : B ⊆F ϵ minimal midly Generalized-C(X)}. If B⊆F ϵ minimal midly Generalized-C(X), then minimal midly Generalized-cl(B) ⊆ F. since A⊆B, A⊆B ⊆ Fϵ minimal midly Generalized-C(X), we have minimal midly Generalized-cl(A)⊆F, minimal midly Generalized-cl(A)⊆∩{F:B⊆F ϵminimal midly Generalized-C(X)} = minimal midly Generalized-cl(B).Therefore midly Generalized-cl(A)⊆minimal midly Generalized-cl(B).

(v)Let A be any subset of X by definition of minimal midly Generalized-closure, minimal midly Generalized-cl(A)= ∩{F:A ⊆F ϵminimal midly Generalized-C(X)}. If A ⊆ F ϵ minimal midly Generalized-C(X) then minimal midly Generalized-cl(A) ⊆ F, since F is minimal midly Generalized-closed set containing minimal midly Generalized-cl(A) by (iii) minimal midly Generalized-cl(midly Generalized-cl(A)⊆F; Hence midly Generalized-cl(minimal midly Generalized-cl(A))=∩{F:A⊆Fϵminimal midly Generalized-C(X)}=minimal midly Generalized-cl(A).Therefore; minimal midly Generalized-cl(minimal midly Generalized-cl(A)) = minimal midly Generalized-cl(A).

(Vi) Let A and B be subsets of X, Clearly A⊆A∪B , B⊆A∪B from (iv) minimal midly Generalized-cl(A) ⊆ minimal midly Generalized-cl(A∪B),minimal midly Generalized-cl(B)⊆minimal midly Generalized-cl(A∪B);hence, minimal midly Generalized-cl(A)∪ minimal midly Generalized-cl(B) ⊆ minimal midly Generalized-cl(A∪B)……..(1)

Now we have to prove that minimal midly Generalized-cl($A\cup B$) $\subseteq$minimal midly Generalized-cl(A)$\cup$ minimal midly Generalized-cl(B).Suppose x$\notin$minimal midly Generalized-cl(A)$\cup$ minimal midly Generalized-cl(B) then $\exists$minimal midly Generalized-closed set A_1 and B_1 with $A\subseteq A_1$, $B\subseteq B_1$ and $x\notin A_1\cup B_1$. We have $A\cup B\subseteq A_1\cup B_1$ and $A_1\cup B_1$ is the minimal midly Generalized-closed set(we know that union of two minimal midly Generalized closed subsets of X is minimal midly Generalized closed set in X) such that $x\notin A_1\cup B_1$.Thus x$\notin$minimal midly Generalized-cl($A\cup B$) hence minimal midly Generalized-cl($A\cup B$)$\subseteq$minimal midly Generalized-cl(A)$\cup$ minimal midly Generalized-cl(B)------(2).From (1) and (2) we have minimal midly Generalized-cl($A\cup B$)=minimal midly Generalized-cl(A)$\cup$minimal midly Generalized-cl(B).

Theorem 3.3: If A $\subseteq$X is minimal midly Generalized-closed set then minimal midly Generalized-cl(A) =A

Proof : Let A be minimal midly Generalized-closed subset of X. we know that A $\subseteq$minimal midly Generalized-cl(A) –(1). Also $A\subseteq A$ and A is minimal midly Generalized-closed set by theorem3.2 (iii) minimal midly Generalized-cl(A)$\subseteq$A –(2) . Hence minimal midly Generalized-cl(A) =A

Example3.4: Let X={a,b,c,d}, τ={X,ϕ,{a},{c,d},{a,c,d}}. A={a,c} minimal midly Generalized-cl(A)={a,c}=A, then A is not minimal midly Generalized-closed set.

Theorem 3.5: If Aand B are subsets of space X then minimal midly Generalized-cl($A\cap B$)$\subseteq$minimal midly Generalized-cl(A)$\cap$minimal midly Generalized-cl(B)

Proof: Let A and B be subsets of X, Clearly $A\cap B\subseteq A$, $A\cap B\subseteq B$ by theorem3.2(iv) minimal midly Generalized-cl($A\cap B$)$\subseteq$minimal midly Generalized-cl(A),Minimal midly Generalized-cl($A\cap B$)$\subseteq$minimal midly Generalized-cl(B);hence minimal midly Generalized-cl($A\cap B$) $\subseteq$minimal midly Generalized-cl(A)$\cap$minimal midly Generalized-cl(B).

Remark 3.6:In-general;minimal midly Generalized-cl(A)$\cap$minimal midly Generalized-cl(B)$\nsubseteq$minimal midly Generalized-cl($A\cap B$) as seen from the following example.

Example3.7: Consider X={a,b,c,d},τ ={X,ϕ,{a},{b},{a,b},{a,b,c}}, A={b,c} , B={c,d},$A\cap B$={c},minimal midly Generalized-cl(A)={b,c,d},minimal midly Generalized-cl(B)={c,d},minimal midly Generalized-cl($A\cap B$)={c} and minimal midly Generalized-cl(A)$\cap$minimal midly Generalized-cl(B)={c,d}.Therefore minimal midly Generalized-cl(A)$\cap$minimal midly Generalized-cl(B) $\nsubseteq$minimal midly Generalized-cl($A\cap B$).

Theorem3.8: For an x ϵX, xϵminimal midly Generalized-cl(A) if and if A∩V≠ ϕ for every minimal midly Generalized-open set V containing x.

Proof: Let xϵminimal midly Generalized-cl(A).To prove A∩V ≠ ϕ for every minimal midly Generalized-open set V containing x by contradiction.Suppose ∃minimal midly Generalized-open set V containing x s.t A∩V = ϕ.Then A ⊆X-V , X-V is minimal midly Generalized-closed set , minimal midly Generalized-cl(A)⊆ X-V. This shows that x∉minimal midly Generalized-cl(A) which is contradiction . Hence A∩V ≠ ϕ for every minimal midly Generalized-open set V containing x.

Conversely:

Let A∩V ≠ ϕ for every minimal midly Generalized-open set V containing x. To prove xϵminimal midly Generalized-cl(A). we prove the result by contradiction. Suppose x∉minimal midly Generalized-cl(A) then there exist a minimal midly Generalized-closed subset Fcontaining A s.t x ∉ F. Then xϵX-F is minimal midly Generalized-open.Also (X-F)∩A=ϕ which is contradiction. Hence xϵminimal midly Generalizedcl(A).

Theorem 3.9: If Ais subset of space X then

(i) minimal midly Generalized-cl(A) ⊆ cl(A)
(ii) minimal midly Generalized-cl(A) ⊆pcl(A)

Proof: (i) Let A besubset of space X by definition of Closure Cl(A)=⊆∩{F : A ⊆F ϵ C(X)}.If A⊆FϵC(X) then A⊆F ϵminimal midly Generalized-C(X) because every closed set is minimal midly Generalized-closed that is minimal midly Generalized-cl(A)⊆F therefore minimal midly Generalized-cl(A) ⊆∩{F : A⊆F ϵ C(X)} = cl(A). Hence minimal midly Generalized-cl(A) ⊆ cl(A).

(ii) Let A be subset of space X by definition of p-closure pcl(A)= ⊆∩{F : A ⊆F ϵ p-C(X)}.If A⊆ F ϵ p-C(X) then A⊆F ϵminimal midly Generalized-C(X) because every p-closed set is minimal midly Generalized-closed that is minimal midly Generalized-cl(A)⊆F .therefore minimal midly Generalized-cl(A) ⊆∩{F : A⊆F ϵ p-C(X)} = pcl(A). Hence minimal midly Generalized-cl(A) ⊆ p-cl(A)

Remark 3.10:Containment relation in the above theorem 3.9 may be proper as seen from following example**.**

Example 3.11: Let X={a,b,c,d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}},A={a} cl(A)={a,c,d} . minimal midly Generalized-cl(A)={a,d}, pcl(A)={a,c,d} It follows that minimal midly Generalized-cl(A)⊂cl(A) and minimal midly Generalized-cl(A) ⊂ pcl(A)

Theorem 3.12 : If A is subset of space X then gpr-cl(A)⊆minimal midly Generalized-cl(A) where gpr-cl(A)= ∩{F : A ⊆F ϵ GPR-C(X)}

Proof: Let A be a subset of X by definition of minimal midly Generalized-closure, minimal midly Generalized-cl(A)=∩{F : A ⊆F ϵminimal midly Generalized-C(X)} If A⊆F ϵminimal midly Generalized-C(X) then A⊆F ϵ GPR-C(X), because every minimal midly Generalized-closed is gpr-closed i.egpr-cl(A) ⊆F therefore gpr-cl(A) ⊆∩{F : A ⊆F ϵminimal midly Generalized-C(X)}= minimal midly Generalized-cl(A). Hence gpr-cl(A) ⊆minimal midly Generalized-cl(A).

Theorem3.13: Minimal midly Generalizedclosure is a Kuratowski closure operator on a space X.
Proof: Let A and B be the subsets space X.

(i) minimal midly Generalized-cl(X)= X , minimal midly Generalized-cl(ϕ)=ϕ
(ii) A ⊆minimal midly Generalized-cl(A)
(iii) minimal midly Generalized-cl(A)= minimal midly Generalized-cl (minimal midly Generalized-cl(A))
(iv)minimal midly Generalized-cl(A∪B) = Minimal midly Generalized-cl(A)∪minimal midly Generalized-cl(B) by theorem3.2
Hence minimal midly Generalized-closure is a Kuratowski closure operator on a space X.

Definition3.14: For a subset A of (X,τ) , minimal midly Generalized-interior of A is denoted by minimal midly Generalized-int(A) and defined as minimal midly Generalized-int(A)= ∪{G:G⊆A and G is minimal midly Generalized-open in X} or ∪{G: G⊆A and G ϵminimal midly Generalized-O(X)}i.e minimal midly Generalized-int(A) is the union of all minimal midly Generalized-open set contained in A.

Theorem 3.15: Let A and B be subset of space X then

(i) Minimal midly Generalized-int(X) = X ,minimal midly Generalized-int(ϕ)=ϕ

(ii) Minimal midly Generalized-int(A)⊆ A

(iii) If B is any minimal midly Generalized-open set contained in A, then B⊆minimal midly Generalized-int(A)

(iv) If$A\subseteq B$ then minimal midly Generalized-int(A)$\subseteq$minimal midly Generalized-int(B)

(v) Minimal midly Generalized-int(A)= minimal midly Generalized-int(minimal midly Generalized-int(A))

(vi) Minimal midly Generalized-int($A\cap B$)=minimal midly Generalized-int(A)$\cap$minimal midly Generalized-int(B)

Proof :(i) and (ii) by definition of minimal midly Generalized-interior of A, it is obvious.

(iii) Let B be any minimal midly Generalized-open set s.t $B\subseteq A$. Let $x \in B$, B is an minimal midly Generalized-open set contained in A, x is an minimal midly Generalized-interior of A i.e. x $\in$minimal midly Generalized-int(A). Hence B $\subseteq$minimal midly Generalized-int(A).

(iv) ,(v), (vi) similar proof as theorem 3.2 and definition of minimal midly Generalized-interior.

Theorem 3.16: If a subset A of X is minimal midly Generalized-open then minimal midly Generalized-int(A) = A

Proof: Let A be minimal midly Generalized-open subset of X. We know that minimal midly Generalized-int(A) $\subseteq$ A –(1) Also A is minimal midly Generalized-open set contained in A from Theorem 3.15(iii)A $\subseteq$minimal midly Generalized-int(A) --(2).Hence From(1) and(2) minimal midly Generalized-int(A) = A.

Theorem 3.17: If A and B are subsets of space X then minimal midly Generalized-int(A)$\cup$minimal midly Generalized-int(B)$\subseteq$minimal midly Generalized-int($A\cup B$)

Proof: We know that A $\subseteq A\cup B$ and $B\subseteq A\cup B$,We have Theorem 3.15(iv)minimal midly Generalized-int(A)$\subseteq$minimal midly Generalized-int($A\cup B$) and minimal midly Generalized-int(B)$\subseteq$minimal midly Generalized-int($A\cup B$). This implies that minimal midly Generalized-int(A)$\cup$minimal midly Generalized-int(B)$\subseteq A\cup B$

Remark 3.18: The converse of the above theorem need not be true as seen from the following example.

Example3.19: Let X = {a,b,c,d} andτ= {X,ϕ,{a},{b},{a,b},{a,b,c}},A={b,c}, B={a,d},$A\cup B$={a,b,c,d},minimal midly Generalized-int(A)={b,c},minimal midly Generalized-int(B)={a},minimal midly Generalized-int($A\cup B$)=X,minimal midly Generalized-int(A) $\cup$minimal midly Generalized-int(B)= {a,b,c}; therefore minimal midly Generalized-int($A\cup$ B) $\not\subseteq$ minimal midly Generalized-int(A)$\cup$ minimal midly Generalized-int(B).

Theorem 3.20 If A is a subset of X then (i) int(A) $\subseteq$minimal midly Generalized-int(A)(ii) p-int(A) $\subseteq$minimal midly Generalized-int(A).

Proof:(i) Let A be a subset of a space X. Let x ϵint(A) => x ϵ ∪{G : G is open , G ⊆ A}=>∃ an open set G s.t.x ϵ G ⊆ A =>∃ an minimal midly Generalized-open set G s.t.x ϵ G ⊆ A, as every open set is an minimal midly Generalized-open set in X=> x ϵ ∪{G : G is minimal midly Generalized-open set in X}=> x ϵminimal midly Generalized-int(A). Thus x ϵint(A) => x ϵminimal midly Generalized-int(A).Hence int(A)⊆minimal midly Generalized-int(A).

(ii) Let A be a subset of a space X. Let x ϵ p − int(A) => x ϵ ∪{G : G is p-open , G ⊆ A}=>∃ an p-open set G s.t.x ϵ G ⊆ A =>∃ an minimal midly Generalized-open set G s.t.x ϵ G ⊆ A, as every p-open set is an minimal midly Generalized-open set in X=> x ϵ ∪{G : G is minimal midly Generalized-open set in X}=> x ϵminimal midly Generalized-int(A). Thus x ϵ p-int(A)
=>x ϵ minimal midly Generalized-int(A). Hence p-int(A)⊆minimal midly Generalized-int(A).

Remark 3.21: Containment relation in the above theorem may be proper as seen from the followingexample

Example 3.22: Let X={a,b,c,d} τ={X,ϕ,{a},{b},{a,b},{a,b,c}};A={b,c}, Int(A)={b},p-int(A)={b}, Minimal midly Generalized-int(A)={b,c}.Therefore int(A)⊆minimal midly Generalized-int(A) and p-int(A)⊆minimal midly Generalized-int(A)

Theorem 3.23: If A issubset of X, then minimal midly Generalized-int(A)⊆gpr-int(A), where gpr-int(A) is given bygpr-int(A) = ∪{G ⊆ X : G is gpr-open , G ⊆ A}
Proof: Let A be a subset of a space X. Let x ϵminimal midly Generalized −int(A) => x ϵ ∪{G : G is minimal midly Generalized-open , G ⊆ A}=>∃ an minimal midly Generalized-open set G s.t.x ϵ G ⊆ A, as every minimal midly Generalized-open set is an gpr-open set in X=> x ϵ ∪{G : G is gpr-open , G ⊆ A }=> xϵgpr-int(A). Thus xϵ minimal midly Generalized-int(A) => xϵgpr-int(A).Hence minimal midly Generalized-int(A)⊆gpr-int(A).

Theorem 3.24: For any subset A of X
(i) X – minimal midly Generalized-int(A) = minimal midly Generalized-cl(X – A)
(ii) minimal midly Generalized-int(A) = X – minimal midly Generalized-cl(X – A)
(iii) minimal midly Generalized-cl(A) = X –minimal midly Generalized-int(X – A)
(iv)X – minimal midly Generalized-cl(A) = minimal midly Generalized-int(X – A)

Proof:xϵX-minimal midly Generalized-int(A) then x is not inminimal midly Generalized-int(A)i.e. everyminimal midly Generalized-open set G containing x s.t. G⊈A . This implies every minimal midly Generalized open set G containing x intersects (X–A) i.e. $G \cap (X - A) \neq \phi$. Then by theorem 3.8xϵ minimal midly Generalized-cl(X-A) therefore X-minimal midly Generalized-int(A)⊆minimal midly Generalized-cl (X-A)–(1) and Let, x ∈minimal midly Generalized-cl(X-A),then every x∈minimal midly Generalized open set ,G containing x intersects X – A i.e. $G \cap (X - A) \neq \phi$, i.e. everyminimal midly Generalized-open G containing x s.t. $G \subseteq A$.

Then by definition 3.14, x is not in minimal midly Generalized-int(A), i.e. xϵX-minimal midly Generalized-int(A) and so – minimal midly Generalized-cl (X-A) $\subseteq$ X- minimal midly Generalized-int(A)- (2) .Thus X– minimal midly Generalized-int(A) = minimal midly Generalized-cl(X – A) .

(ii)Follows by taking complements in (i).

(iii) Follows by replacing A by X-A in (i)

(iv) Follows by taking complements in(iii).

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